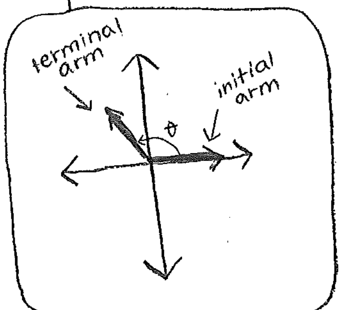
Chapter 2: Trigonometry

Angles in Standard Position

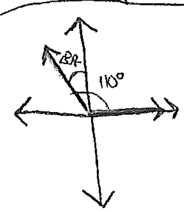
vertex is at the origin



closest x-axis

acute

Reference Angles

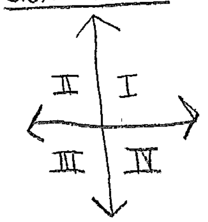


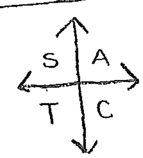
$\angle\theta = 110°$

$\angle\theta_R = 110 - 90$

$\angle\theta_R = 20°$

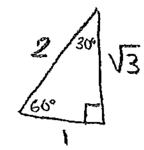
QUADRANTS



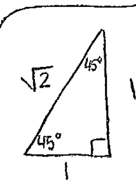


Special Right Triangles

30°-60°-90°



45°-45°-90°



Sine, Cosine, Tangent

SOH CAH TOA

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

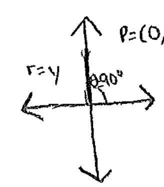
to find the hypotenuse

Pythagorean Theorem

$a^2 + b^2 = c^2$

terminal arm lies on one of the axes

Quadrantal Angles



$\tan 90 = \frac{y}{x}$

$\tan 90 = \frac{y}{0}$

$\tan 90 =$ undefined

$\sin 90 = \frac{y}{r}$

$\sin 90 = \frac{y}{y}$

$\sin 90 = 1$

$\cos 90 = \frac{x}{r}$

$\cos 90 = \frac{0}{y}$

$\cos 90 = \emptyset$

sides of a triangle are proportional to signs of opposite angles

Sine law

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

use only if you know all 3 sides or 2 sides and 1 angle

Cosine Law

SAS $\xrightarrow{\text{find}}$ side

SSS $\xrightarrow{\text{find}}$ θ

$c^2 = a^2 + b^2 - 2ab\cos C$

OR

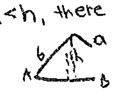
$\cos A = \frac{b^2 + c^2\ a^2}{2bc}$

comparing a, b, and height

Ambiguous Cases

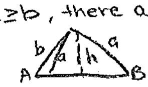
If $a<h$, there is no solution

ex//



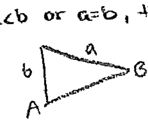
If $a \geq b$, there are two solutions

ex//



If $a<b$ or $a=b$, there is 1 solution

ex//



Chapter 3: Quadratic Functions

Parabola

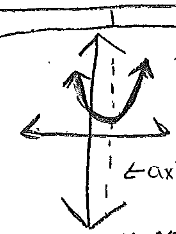
Vertex

lowest point on graph if it opens ↑
highest point on graph if it opens ↓

min. if parabola opens ↑
max. if parabola opens ↓

Minimum/Maximum Values — y-coordinate of vertex

a line through the vertex that divides the graph of a quadratic function into 2 halves — **Axis of Symmetry**



→ vertex $(3, 1)$
axis of symmetry $x = 3$
← axis of symmetry
defined by x-coordinate of a vertex

Vertex Form

$f(x) = a(x-p)^2 + q$
vertex (p, q)
$a \neq 0$

Standard Form

$f(x) = ax^2 + bx + c$
$y = ax^2 + bx + c$

X-intercept

when a line crosses through the x-axis
$(x, 0)$

Completing the Square

used to write a quadratic polynomial in vertex form

STANDARD $y = x^2 - 8x + 5$	
$y = (x^2 - 8x) + 5$	Group the first 2 terms
$y = (x^2 - 8x + 16 - 16) + 5$	Add and subtract the square of half the coefficient of the x-term.
$y = (x^2 - 8x + 16) - 16 + 5$	Group perfect square trinomial.
$y = (x-4)^2 - 16 + 5$	Rewrite as square of a binomial.
$y = (x-4)^2 - 11$ VERTEX	Simplify.

$y = 5x^2 - 30x + 7$
$y = 5(x^2 - 6x) + 7$
$y = 5(x^2 - 6x + 9 - 9) + 7$
$y = 5[(x^2 - 6x + 9) - 9] + 7$
$y = 5(x-3)^2 - 45 + 7$
$y = 5(x-3)^2 - 38$

Chapter 4: Quadratic Equations

an equation with the standard form
$ax^2 + bx + c = \varnothing$
$a \neq \varnothing$

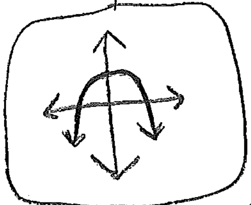
the solution(s) of an equation

Root(s) of an equation

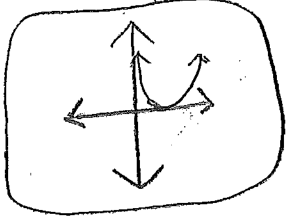
Determine x-intercepts or zeroes

FACTOR

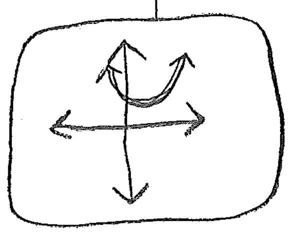
2 Real Roots



1 Real Root



No Real Roots



$ax^2+bx+c,\ a \neq \varnothing$

Factoring Quadratic Expressions

Factor out common factors
ex// $4x^2-2x-12 = 2(2x^2-x-6)$
$= 2(2x^2-4x+3x-6)$
$= 2[2x(x-2)+3(x-2)]$
$= 2(x-2)(2x+3)$

sum/product

Solve Quadratic Equations by Factoring

$3x^2-2x-5=\varnothing$
$(3x-5)(x+1)=\varnothing$
$3x-5=\varnothing$ or $x+1=\varnothing$
$x=\frac{5}{3}$ $\quad x=-1$
The roots are $\frac{5}{3}$ & -1.

Verify by checking Left Side = Right Side

Solve Quadratic Equations by completing the square

$x^2-21=-10x$
$x^2+10x=21$
$x^2+10x+25=21+25$
$(x+5)^2=46$
$x+5=\pm\sqrt{46}$

extraneous root: A # obtained in solving an equation, which does not satisfy the equation

to determine roots of quadratic equation

Quadratic Formula
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

used to determine nature of roots for a quadratic equation

discriminant
b^2-4ac

if $x+3=0$
$x=-3$
↑Root

$3x^2+5x-2=\varnothing$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-5\pm\sqrt{5^2-4(3)(-2)}}{2(3)}$
$x=\frac{-5\pm\sqrt{25+24}}{6}$
$x=\frac{-5\pm\sqrt{49}}{6}$
$x=\frac{-5\pm7}{6}$

Roots
$x=\frac{-5+7}{6}$
$x=\frac{1}{3}$
$x=\frac{-5-7}{6}$
$x=\frac{-12}{6}$
$x=-2$

Chapter 5 : Radical Expressions and Equations

radicals w/ the same radicand & index

when adding and subtracting radicals, only like radicals can be combined.

Like Radicals
ex// $5\sqrt{7}$ & $-\sqrt{7}$
$2\sqrt[3]{3}$ & $8\sqrt[3]{3}$

If the radical has an even index, the radicand must be non-negative.

converting mixed radicals to entire radicals

Example
$$7\sqrt{2} = \sqrt{7^2}(\sqrt{2})$$
$$= \sqrt{7^2(2)}$$
$$= \sqrt{49(2)}$$
$$= \sqrt{98}$$

Radicals in simplest form do not contain fractions or factors which may be removed. They are also not part of the denominator of a fraction

Express Entire Radicals as Mixed Radicals

Example
$$\sqrt{200} = \sqrt{100(2)}$$
$$= \sqrt{100}(\sqrt{2})$$
$$= 10\sqrt{2}$$

greatest perfect-square factor

can only multiply radicals if they have the same index

Multiplying Radicals

Multiply the coefficients and multiply the radicands.
$$(2\sqrt{7})(4\sqrt{75}) = (2)(4)\sqrt{(7)(75)}$$
$$= 8\sqrt{525}$$
$$= 8\sqrt{(25)(21)}$$
$$= 8(5)\sqrt{21}$$
$$= 40\sqrt{21}$$

Adding & Subtracting Radicals
$$\sqrt{50} + 3\sqrt{2}$$
$$\sqrt{25(2)} + 3\sqrt{2}$$
$$5\sqrt{2} + 3\sqrt{2}$$
$$= 8\sqrt{2}$$

$(\sqrt{2})(\sqrt{2}) = 2$
$(\sqrt{3})(\sqrt{3}) = 3$

convert to rational # w/out changing value of expression

Rationalize Denominators

If the radical is in the denominator, both the numerator and denominator must be multiplied by a quantity that will produce a rational denominator.
ex// $\dfrac{5}{2\sqrt{3}} \rightarrow \dfrac{5}{2\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)$
$$= \frac{5\sqrt{3}}{2\sqrt{3}(\sqrt{3})}$$
$$= \frac{5\sqrt{3}}{6}$$

multiply by a conjugate

Rationalizing Binomial Denom.
$$\frac{5\sqrt{3}}{4-\sqrt{6}} = \left(\frac{5\sqrt{3}}{4-\sqrt{6}}\right)\left(\frac{4+\sqrt{6}}{4+\sqrt{6}}\right)$$
$$= \frac{20\sqrt{3} + 5\sqrt{18}}{4^2 - (\sqrt{6})^2}$$
$$= \frac{20\sqrt{3} + 5\sqrt{9(2)}}{16-6}$$
$$= \frac{20\sqrt{3} + 15\sqrt{2}}{10}$$
$$= \frac{4\sqrt{3} + 3\sqrt{2}}{2}$$

use for dividing radicals
$$\frac{4\sqrt{5n}}{3\sqrt{2}}\frac{(\sqrt{2})}{(\sqrt{2})}$$
$$= \frac{4\sqrt{10n}}{3(2)}$$
$$= \frac{2\sqrt{10n}}{3}$$

conjugate
2 binomial factors whose product is the difference of 2 squares*

Radical Equations

square both sides

1 Radical Term
$$5 + \sqrt{2x-1} = 12$$
$$\sqrt{2x-1} = 7$$
$$(\sqrt{2x-1})^2 = (7)^2$$
$$2x - 1 = 49$$
$$2x = 50$$
$$x = 25$$

Remember to VERIFY

Isolate 1 radical
square both sides

2 Radicals
$$7 + \sqrt{3x} = \sqrt{5x+4} + 5$$
$$2 + \sqrt{3x} = \sqrt{5x+4}$$
$$(2+\sqrt{3x})^2 = (\sqrt{5x+4})^2$$
$$4 + 4\sqrt{3x} + 3x = 5x + 4$$
$$4\sqrt{3x} = 2x$$
$$(4\sqrt{3x})^2 = (2x)^2$$
$$16(3x) = 4x^2$$
$$0 = 4x^2 - 48x$$
$$0 = 4x(x-12)$$
$$4x = 0 \quad \text{or} \quad x - 12 = 0$$
$$x = 0 \qquad x = 12$$

Chapter 6 : Rational Expressions and Equations

Rational Expression

An algebraic fraction w/ a numerator and denominator that are polynomials

*Non Permissable Values

Any value for a variable that makes an expression undefined. In a rational expression, a value that makes the denominator Ø *

Equivalent Expressions

$$\left(\frac{7s}{s-2}\right)\left(\frac{s}{s}\right) = \frac{(7s)(s)}{s(s-2)}$$

$$= \frac{7s^2}{s(s-2)}$$

$s \neq 0, 2$

Simplifying Rational Expressions

$$\frac{9}{12} = \frac{(\cancel{3})(3)}{(\cancel{3})(4)}$$

$$= \frac{3}{4}$$

$$\frac{m^3t}{m^2t^4} = \frac{(\cancel{m^2})(m)(\cancel{t})}{(\cancel{m^2})(\cancel{t})(t^3)}$$

$$= \frac{m}{t^3}$$

$m \neq Ø,\ t \neq Ø$

Multiplying Rational Expressions

$$\left(\frac{5}{8}\right)\left(\frac{4}{15}\right) = \frac{(5)(4)}{(8)(15)}$$

$$= \frac{(5)(4)}{(2)(4)(3)(5)}$$

$$= \frac{1}{6}$$

$$\frac{a^2-a-12}{a^2-9} \times \frac{a^2-4a+3}{a^2-4a}$$

$$= \frac{(a-4)(a+3)}{(a-3)(a+3)} \times \frac{(a-3)(a-1)}{a(a-4)}$$

$$= \frac{(a-4)(a+3)(a-3)(a-1)}{(a-3)(a+3)(a)(a-4)}$$

$$= \frac{a-1}{a} \qquad a \neq 0, -3, 3, 4$$

Dividing Rational Expressions

1) Use Common Denominator

$$\frac{5}{3} \div \frac{1}{6} = \frac{10}{6} \div \frac{1}{6}$$

$$= \frac{10}{1}$$

$$= 10$$

2) Multiply by Reciprocal

$$\frac{5}{3} \div \frac{1}{6} = \frac{5}{3} \times \frac{6}{1}$$

$$= 10$$

Adding and Subtracting Rational Expressions

1) Same Denominators
- Add or subtract the numerators, write w/ new numerator over common denominator

2) Denominators are Different
- write equivalent fractions w/ the same denominators

$$\frac{2a}{b} - \frac{a-1}{b}$$

$$\frac{2a-(a-1)}{b}$$

$$\frac{2a-a+1}{b} \qquad b \neq Ø$$

$$\frac{a+1}{b}$$

$$\frac{10}{3x-12} - \frac{3}{x-4} = \frac{10}{3(x-4)} - \frac{3}{x-4}$$

$$= \frac{10}{3(x-4)} - \frac{3(3)}{(x-4)(3)}$$

$$= \frac{10-9}{3(x-4)}$$

$$= \frac{1}{3(x-4)} \qquad x \neq 4$$

Rational Equations

$$\frac{2}{z^2-4} + \frac{10}{6z+12} = \frac{1}{z-2}$$

$$\frac{2}{(z-2)(z+2)} + \frac{10}{6(z+2)} = \frac{1}{z-2}$$

$$(z-2)(z+2)(6)\left(\frac{2}{(z-2)(z+2)} + \frac{10}{6(z+2)}\right) = (z-2)(z+6)(6)\left(\frac{1}{z-2}\right)$$

$$6(2) + (z-2)(10) = (z+2)(6)$$

$$12 + 10z - 20 = 6z + 12$$

$$4z = 20$$

$$z = 5$$